DEPARTMENT OF MATHEMATICS & ASTRONOMY
FACULTY OF SCIENCE / FACULTY OF ARTS
UNIVERSITY OF LUCKNOW

Syllabus for M.A./M.Sc. Course under Semester System

SEMESTER -I

There will be FIVE compulsory core papers each having 70 marks. In addition there will be a Mid-term test in each paper and it will carry 30 marks.

Paper I: Topology I

Unit I
Countable and uncountable sets, Infinite sets and the axiom of choice, Cardinal numbers and its arithmetic, Schroeder-Bernstein theorem, Cantor's Theorem and Cantor's continuum hypothesis, Zorn's Lemma, Well ordering principle.

Unit II
Definition and examples of topological spaces, Closed sets, Closure, Dense subsets, Neighbourhoods, Interior, exterior and boundary, Accumulation points and derived sets, Bases and subbases, Subspaces and relative topology.

Unit III
Alternative methods of defining a topology in terms of Kuratoivski closure operator, interior operator and neighbourhood systems, Continuous functions and homeomorphism,First & Second countable spaces, Lindeloff theorem and separable spaces and their relationships.

Unit IV
Separation axioms T_0, T_1, T_2, Nets and filters, Topology and convergence of nets. Hausdorffness and nets, Filters and their convergence, Ultra filters, Canonical way of converting nets to filters and vice-versa.

Books Recommended:

1. GF Simmons: Introduction to Topology and Modern Analysis, Mc Graw Hill, 1963.
2. JL Kelly: Topology, Von Nostrand Reinhold Co. New York, 1995.

Paper II: Advanced Algebra

Unit I
Series of groups, Schreier Theorem, Jordan Holder Theorem, Solvable groups, Nilpotent groups, Insolvability of S_n for $n>5$, finite Abelian groups, fundamental theorem of finite Abelian group.

Unit II
Field extensions: Finite extension, Finitely generated extension, Algebraic extension, Simple extension, Transcendental Extension, Finite Field.

Unit III
Splitting field, Algebraically closed field, Normal extension, Separable extension, Primitive Element Theorem.

Unit IV
Galois Theory- Galois group, Galois extension, Fundamental Theorem of Galois Theory, Artin's Theorem, Fundamental Theorem of Algebra(Algebraic Proof)

Books Recommended :

1. Serge Lang: Algebra, Addison Wesley
2. V.Sahai & V.Bist: Algebra, Second edition, Narosa.

Paper III: Differential Geometry of Manifolds

Unit I
Definition and examples of differentiable manifolds, Tangent Spaces, Vector fields, Jacobian map, Distributions, Hypersurface of R^n,

Unit II
Standard connection on R^n, Covariant derivative, Sphere map, Weerigarten map, Gauss equation, the Gauss curvature equation and Coddazi-Mainardi equations.

Unit III
Invariant view point cortan view point coordinate view point, Difference Tensor of two connections, Torsion and curvature tensors.

Unit IV
Riemannian Manifolds, Length and distance in Riemannian manifolds, Riemanian connection and curvature, Curves in Riemannian manifolds, Submanifolds.

Books Recommended:

1. NJ Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. Y Matsushima: Differentiable Manifolds

Paper IV: Integral Equations and Partial Differential Equations

Unit I
Linear Integral Equations-Definition and Classification of conditions, Special kinds of Kernels, Eigen values and eigen functions, Convolution integral, Inner product, Integral Equations with separable Kernels, Reduction to a system of algebraic equations,

Unit II

Fredholm alternative, Fredholm Theorem, Fredholm alternative theorem, Approximate method, Method of successive approximations- Iterative scheme, solution of Fredholm and Volterra integral equation. Results about resolvent Kernel.

Unit III

Singular integral equation – Abel integral equation, General forms of Abel Singular integral equation, Weakly singular kernel Cauchy principal value of integrals, Hilbert kernel, Hilbert formula, Solution of Hilbert type singular integral equation of first and second kind. Fundamental properties of symmetric kernels.

Unit IV

Cauchy's method of characteristic, Cauchy's problem for Homogenous wave equation, Properties of Harmonic function, Energy equation, Methods of separation of variable for solving Laplace, wave and diffusion equations.

Books Recommended :

1. I. N. Sneddon: Elements of Partial Differential Equations, Mc Graw Hill, 1988.
2. Tyn Myint-U: Partial Differntial Equations of Mathematical Physics, Elseveir Publications.

Paper V: Real Analysis

Unit I

Algebra of sets, outer Measure, Measurable Sets and Lebesgue measure, non-measurable sets, measurable functions.

Unit II

The Lebesgue inegration of bounded function over a set of finite measure, the integral of a non-negative functions, The general Lebesgue integral.

Unit III

The four derivatives, differentiation of monotone functions, functions of bounded variation, Lebesgue differentiation theorem, Differentiation of an integral. Absolute continuity.

Unit IV

Inequalities and the L_p Spaces, The L_p Spaces, convex functions, Jensen's ineauality, the inequalities of Holder and Minkowski, completeness of $L_p(\mu)$. Convergence in Measure, almost uniform convergence.

Books Recommended:

1. H.L. Royden: Real Analysis
2. G.de Barra: Measure Theory and Integration

Syllabus for M.A./M.Sc. (Mathematics)Semester I (Elective Paper E1)

Elective Paper E1: Discrete Mathematics

Unit I

Mathematical Logic, Statement calculus: Propositional logic, Logic operators or connectives, Well formed formula (wff), Construction of truth-table for a formula, Equivalence of formulas, Tautology, Contradiction argument, Valid argument, Proving validity by truth-table methods, Inference theory of statement calculus, Minimal sets of logic operators. Predicate calculus: Statement function and statement, Proving validity by the deduction method, Inference rules, Proving validity by the method of contradiction..

Unit II

Lattice theory and Boolean algebra Lattice Theory: partial order relation, Partially ordered set, Totally ordered set, Hasse Diagrams, Lattice, Lattice as an algebraic system, Bounded lattice, Complemented lattice, Distributive lattice, Direct product, Lattice homomorphism, Boolean algebra: Boolean functions, Principle of duality, Boolean function minimization, Sum of products and product of sums form, Normal forms, Conversion of normal forms into principal normal forms, Boolean function minimization, Logic circuits, Designing logic circuits.

Unit III

Automata theory, Finite state automaton, Types of automaton, Deterministic finite state automaton, Non deterministic finite state automaton, Non deterministic finite-state automaton with ε, Equivalence of NFA and DFA, Equivalence of NFA and NFA-ε, Equivalence of NFA-ε and DFA, Finite state Machines: Moore and Mealy machine, and their conversion, Turing machine.

Unit IV

Grammars and Languages, Regular language, Regular expression Equivalence of Regular language and finite state automaton, Grammar: Context-free and Context-sensitive grammar, LR Grammar: Contruction of LR(0) parsing table, Construction of LR(1) parsing table, Decision algorithms for CFL.

References :

1. Mendelson, Elliott: Introduction to Mathematical Logic, Chapman & Hall, 1997.
2. John E. Hoprcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Aotomata Theory, Languages and Computation, Pearson Education, 2000.
3. Arnold B. H.: Logic and Boolean Algebra, Prentice Hall, 1962.
4. K. H. Rosen: Discrete Mathematics and its applicatons, MGH 1999.

Syllabus for M.A./M.Sc.(Mathematics) Course under Semester System

SEMESTER-II

Paper I: Topology II

Unit I

Seperation axioms T_3 $T_{31/2}$,T_4 and their basic properties, Urysohn's lemma, Tietze extension theorem. Metric spaces, compactness and its basic properties, Local compactness and one point compactification.

Unit II

Compactness in metric spaces, Bolzano-Weierstrass property, Sequential compactness, countable compactness, equivalence of compactness, Countable compactness, Sequential compactness in metric space. Connected spaces, connectedness on the real line, Components, Locally connected spaces.

Unit III

Tychonoff product topology in terms of standard subbase and its characterization, Projection maps, separation axioms and product spaces, Connectedness and Compactness (Tychonoff theorem) with product spaces, Countability and product spaces. Embedding and metrization, Embedding lemma and Tychonoff embedding. The Urysohn's metrization theorem.

Unit IV

The fundamental group and convering spaces- Homotopy of paths. The fundamental group, covering spaces, The fundamental group of circle and the fundamental theorem of algebra.

Books Recommended:

1. James R Munkres: Topology, A first course, Prentice Hall, New Delhi, 2000
2. GF Simmons: Introduction to Topology and Modern Analysis, Mc Graw Hill, 1963.
3. JL Kelly: Topology, Van Nostrand Reinhold Co. New York, 1995.

Paper II: Module Theory

Unit I

Modules-Definition and examples, simple modules, submodules, Module Homomorphisms, Quotient modules, Direct sum of modules.

Unit II

Exact sequences, Short exact sequence, split exact sequences. Torsion free and torsion modules Free modules- Definition and examples, modules over division rings are free modules.

Unit III
Free modules over PID's, Invariant factor theorem for sub modules, Finitely generated modules over PID, Chain of invariant ideals, Fundamental structure theorem for finitely generated module over a PID,

Unit IV
Projective and injective modules, Divisible group.

Books Recommended :

1. Serge Lang: Algebra, Addison Wesley.
2. V.Sahai & V.Bist,: Algebra, Second Edition, Narosa.

Paper III: Riemannian Manifolds, Lie Algebra and Bundle Theory

Unit I
Sectional Curvature, Schur's Theorem, Geodesic in a Riemannian Manifold, Projective Curvature tensor, Concircular Curvature Tensor, Conformal curvature tensor, Conharmonic curvature tensor, Einstein Manifolds

Unit II
Tensor and forms, Exterior derivative, contraction, Lie derivative, general covariant derivative.

Unit III
Lie groups and Lie algebras with examples, homomorphism, isomorphism, one parameter subgroups and exponential map, The Lie transformations group.

Unit IV
Principal fibre bundle, Linear frame bundle, Associated bundles, tangent bundle.

Books Recommended:

1. NJ Hicks: Notes on Differential Geometry, D. Van Nostrand, 1965.
2. BB Sinha: An introduction to Modern Geometry

Paper IV: Ordinary Differential Equations

Unit I
Linear System- Introduction, properties of linear homogeneous systems, Periodic linear System, Floquet's theorem, Inhomogeneous linear system.

Unit II
System of first order equation:Linear system, Homogenous linear system with constant coefficient, Nonlinear system, Volterra's prey & predator equation, Non Linear equation: Autonomous system. The phase plane & its phenomena, types of critical points & stability.

Unit III
Critical points & stability for linear system, stability by Liapunov's direct method Green function, Construction of Green functions, Strum Liouville systems.

Unit IV
Second order differential equation: Introduction, Preliminary results, Boundedness of solutions, Oscillatory equation, number of zeros, Pruffer's transformation, Strum theorem, Strum's comparison theorem.

Books Recommended :

1. Differential Equation : G.F. Simmons, TMH
2. A coure in Ordinary Differential Equations: B. Rai, D. P. Chaudhary, H. I. Freedman, Narosa Publishing House Chapter IV.

Paper V: Complex Analysis

Unit I
Schwarz's Lemma, Minimum Modulus Theorem, Hadamard'sthree circle theorem, Automorphism of the unit disk, Series representation of analytic functions; Convergence of sequences and series of complex numbers, Absolute convergence, Uniform convergence of sequence of functions, Cauchy criterion, WeierstrassM-test, Analyticconvergence theorem,Absolute and uniform convergence of power series, Integration and differentiation of power series, Radius of convergence.

Unit II
Zeros of holomorphic functions, Open Mapping theorem, Inverse function theorem. Index of a closed path, Meromorphic functions, Argument principle, Rouche'stheorem, Residue at the point at infinity, Indentation around a branch point and the branch cut, Summation of series.

Unit III
Function spaces: Hurwitz theorem, Infinite products, Weierstrassfactorization theorem, Mittag-Leffler'stheorem, Gamma functions and its properties, Riemann's Zeta function.

Unit IV
Uniqueness of direct analytic continuation, Power series method of analytic continuation, Natural boundary, Schwarz's reflection principle, Harmonic Functions, Mean value property for harmonic functions, Hornbeck'sinequality, Canonical products, Poisson formula, Jensen's formula, Poisson-Jensen's formula, Convex functions,Hadamardthree circle theorem as aconvexity theorem, Hadamardfactorization theorem, order of entire functions.

Books recommended:

1. J. V. Deshpande: Complex Analysis
2. E. C. Titchmarsh: Theory of functions
3. John B. Conway: Functions of one complex variables
4. R.V. Churchil & J.W. Brown : Complex Variables and Applications

Syllabus for M.A./M.Sc. (Mathematics) Semester II (Elective Paper E2)

Elective Paper E2: Graph Theory

Unit I

Graph and its terminology, Directed and undirected graph, Multi graph, Simple graph, Complete graph, Weighted graph, Planar and non-planar graph, Regular graph, Graph isomorphism and homeomorphism, Euler's formula, Statement and applications of Kuratowski's theorem, Path factorization of a graph, representing graphs in computer system, Coloring of graph.

Unit II

Graph connectivity, Konigsberg bridge problem, Euleria path and Elerian circuit, Hamiltonian path and Hamiltonian circuit, Shortest path, Dijkstra's algorithm, Paths between the vertices, Path matrix, Warshall's algorithm, Cut point, bridge, cut sets and connectivity, Menger's theorem.

Unit III

Tree and related terminology, spanning tree, Finding minimum spanning tree by Kruskal's algorithm and Prim's algorithm, inorder, preorder, and postorder tree traversals, Binary tree, Expression trees and reverse polish notation (RPN), RPN evaluation by stack.

Unit IV

Flow network, Feasible flows, Multiple sources and multiple sinks, Cutsets in flow network, Relation between flows and cuts, Max flow problem, Max flow min-cut therem, Matching, Covering, Application of networks in Operations Research – CPM/PERT.

References :

1. Graph Theory, Harary, Addison- Wesley 1969
2. Introduction to Graph Theory, D. B. West, Prentice Hall 1996.
3. Graph Theory and Its Applications, Jonathan Gross and Jay Yellan, CRC 1998.

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Syllabus for M.A./M.Sc. Course under Semester System

There will be THREE compulsory core courses and TWO OPTIONAL PAPERS each having 70 marks. In addition there will be a Mid-term test in each course and it will carry 30 marks.

SEMESTER-III

Compulsory Papers

Paper I: Functional Analysis

Unit I
Banach Spaces- the definition and some examples, continuous linear transformations, The Hahn Banach theorem.

Unit II
The natural imbedding of N in N**, the open mapping theorem, the conjugate of an operator.

Unit III
Hilbert spaces- the definition and some simple properties, Orthogonal complements, orthogonal sets, the Conjugate space H*.

Unit IV
The adjoint of an operator, Self adjoint operators, normal and unitary operators, Projections. Finite dimensional spectral theory – Spectrum of an operator, the spectral theorem.

Books :Recommended :
G F Simmons: Introduction to Topology & Modern Analysis (Mc Graw Hill).

Paper II: Structures on even dimensional differentiable manifolds

Unit I
Almost complex manifolds, Nijenhuis tensor, contravariant and covariant analytic vector. Almost Hermite manifold, almost analytic vector fields curvature tensors, Linear conncetions.

Unit II
Kahler manifolds, affine connections, curvature tensors, contravariant almost analytic vectors.

Unit III
Nearly Kahler manifold, curvature identities, Curvature tensors, almost analytic vectors.

Unit IV
Almost Kahler manifolds, analytic vectors conformal transformations, curvature identities.

Books Recommended :

R.S. Mishra, Structures on a differentiable manifold and their applications, Chandrama Prakashan, Allahabad.

Paper III: Fluid Mechanics I

Unit I
Types of fluids, Lagrangian and Eulerian method of describing fluid motion, Motion of Fluid element: Translation, Rotation and Deformation. Stream lines, Path lines and streak lines. Material derivative. Acceleration of a fluid particle in Cartesian, Cylindrical Polar and Spherical Polar Coordinates. Vorticity Vector, Vortex Lines, Rotational and Irrotational motion of fluid, Velocity Potential, Boundary surface, Boundary condition.

Unit II
Reynold transport theorem. Principle of conservation of mass-Equation of continuity (By Lagrangian and Eulerian method. Equation of Continuity in different coordinate systems. Body force and Surface force. Euler's equation of motion-conservation of momentum, Energy Equation, Bernoulli's Equation.

Unit III
Irrotational motion in two dimensions: Stream function, Physical significance of stream function, Complex Velocity Potential. Sources, Sinks, Doublets and their images in two dimension. Milne-Thompson circle theorem. Vortex motion. Complex Potential due to Vortex circulation, Kelvin's theorem on Vortex motion, Blasius Theorem and Kutta-Joukowski Theorem.

Unit IV
Irrotational motion produced by motion of circular cylinders in an infinite mass of liquid, Liquid Streaming past circular cylinder, Kinetic energy of liquid, Motion of sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Axis-Symmetric flow, Stoke's function.

Books Recommended:

1. G. K. Batchelor: An Introduction to Fluid Dynamics.

2. Frank Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi.
3. Z.U.A. Warsi: Fluid Dyanmics, Theoretical and Computational Approaches, C.R.C. Press
4. S.W. Yuan: Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd. New Delhi
5. N. Curle and H J Davies: Modern fluid dynamics

Optional Papers for Semester-III

Paper V: Special Functions

Unit I

The Gamma Functions: Analytic Character, Tannery's theorem, Euler's limit formula, Duplication formula, Eulerian integral of the first kind, Euler's Constant, Canonical product, Asymptotic expansions, Watson's lemma, Asymptotic expansion of r(z) and its range of validity, Asymptotic behavior of |r(x+iy)|, Hankel's contour integral.

Unit II

The Hypergeometric Functions: Solution of homogeneous linear differential equation of order two near an ordinary point and near a regular singularity, Convergence of the series solution near a regular singularity, solutions valid for large value of |z|, solution when the exponent difference is an integer or zero, second-order differential equation with three regular singularity, Hypergeometric equation and its solution, generalized Hypergeometric equation.

Unit III

Integral representation of F(a,b,c,z), value of F(a,b,c;1) when Rl(c-a-b)>0, Analytical continuation of F(a,b;c;z), Barnes's contour integral for F(a,b;c;z), behavior of F(a,b;c;z) near the point at infinity, relation between contiguous hypergeometric functions, generalized hypergeometric function, Confluent hypergeometric function, Asymptotic expansion, Asymptotic expansion of ${}_1F_1(a;b;z)$.

Unit IV

Bessel Functions:Bessel's differential equation and its series solutions, recurrence formulae for $J_v(z)$, Schlafli's contour integral for $J_v(z)$, generating functions for $J_n(z)$ solution of Bessel's equation by Complex integrals, Hankel functions, Connexion between the Bessel and Hankel functions, complete solution of Bessel's equation, Bessel function of the second kind, series for $Y_n(z)$, Asymptotic expansion of the Bessel's functions, Neumann polynomials, Neumann's expansion theorem.

Books recommended:

Theory of function of Complex Variable : E.T. Copson

Paper VIII: Mathematical Biology

Unit I
Continuous population Models for single species: Continuous Growth Models, Insect out break Model: Spruce Budworm, Delay models, Linear Analysis of Delay Population Models: Periodic solutions.

Unit II
Delay Models in Physiology; I Dynamic Diseases, Harvesting a single Natural Population, Population Model with Age Distribution, Simple Discrete Models.

Unit III
Continuous Models for interacting Population : Interaction between species: two species models, definition of stability, community matrix approach, Qualitative behavior of the community matrix, Competition: Lotka-Volterra models, Extension to Lotka_Volterra models, Competition in field experiments, Competition for space, Models for Mutualism.

Unit IV
Predator: Prey interaction: Lotka-Volterra Models, dynamic of the simple Lotka_Volterra models, Role of density dependent in the Prey, Classic laboratory experiment on predator, predation in natural system. Some predator- prey models.

Books recommended:
1. Mathematical Biology : J.D. Murrey.
2. Population Biology : Alan Hastings Concepts and Models , Springer

Paper X: Ordinary Differential Equation

Unit I
The Theory of First Order Equations: Some concepts from real function theory, existence and uniqueness of solutions, Dependence of solutions on initial conditions and on the function.

Unit II
The theory of Linear Differential Equations: The basic existence theorem, Basic theory of the homogeneous linear equation, Linear independence and Fundamental Systems, properties of the homogeneous Linear equation, reduction of order, the non homogeneous equation, the adjoint equation.

Unit III
Systems of Differential Equations: Introduction. Theory of Homogeneous Constant Constant Coefficient Systems, Non-homogeneous Systems, Matrix Formulation, Solving Constant Coefficient Systems in 2D. Examples of the Matrix Method. Eigenvalue Problems, Green's matrix and its properties. Solution of boundary value problem by Green's matrix.

Unit IV

Sturm-Liouville Systems . Eigen functions, Bessel functions, Singular Sturm Liouville systems, Legendre functions boundary value problem for Ordinary differential equation. Solution by Eigenfunction Expansion, and Green's functions, construction of Green's function for Ordinary differential equation.

Books recommended:

1. Differential Equations by S.L.Ross, Blaisdell Publishing Company.
2. Ordinary Differential Equation by R.H.Cole.

Paper XIII: Astronomy and Astrophysics

Prerequisite: This paper is open to students who studied Physics in UG.

Unit I: Solar System and Stars

1. The solar system: Celestial mechanics, Elliptical orbits, Kepler's laws, Earth-moon system, Tidal forces, Precession of earth's axis, Interiors, Atmospheres, Planets: Terrestrial planets & Jovian planets

2. Observational tools: Blackbody radiation, Specific intensity and flux density, Stellar parallax Magnitudes, Colour index, Basic optics and optical telescopes, Radio telescopes Infrared, ultraviolet and X-ray telescopes,

3. Star: Classification, Formation of spectral lines, Hertzsprung-Russell diagram, Atmospheres, Description of the radiation field, Opacities, Radiative transfer, Structure of spectral lines

4. Sun: Interior, Atmosphere, Solar activity, Helioseismology

Unit II: Stellar Structure and Evolution

1. Stellar interiors: Hydrostatic equilibrium, Pressure equation of state, Energy sources Energy transport and convection, Model building, Main sequence

2. Binary stars: Classification, Mass determination, Accretion disks in close binaries, White dwarfs, neutron stars and black holes in binaries

3. Star formation: Interstellar dust and gas, Formation of protostars, Pre and post-main sequence evolution

4. Degenerate remnants of stars: White dwarfs, Chandrasekhar limit, Neutron stars Pulsars

Unit III: High Energy Astrophysics

1. Radiative processes in astrophysics: Synchrotron emission, Energy loss and electron spectrum, Compton scattering, Bremstrahlung, Thermal bremstrahlung

2. Binary stars: White dwarf binaries, Neutron star and black hole binaries, HulseTaylor binary pulsar

3. Accretion discs: Thin & Thick accretion discs, Accretion discs in binaries, Accretion discs in galactic nuclei

Unit IV: Galaxies

1. The Milky Way Galaxy: Distribution of stars, Morphology, Kinematics, Interstellar medium
2. Nature of galaxies: Hubble sequence, Spirals and irregular galaxies, Spiral structure Elliptical galaxies

3. Galactic evolution: Interaction of galaxies, Formation of galaxies

4. Structure of the universe: Extragalactic distance scale, Expansion of the universe, Clusters of galaxies.

Text books:

1. Modern Astrophysics, B. W. Carroll and D. A. Ostlie, Addison-Wesley Publishing Co.
2. Introductory Astronomy & Astrophysics, M. Zeilik and S. A. Gregory, 4th Edition, Saunders College Publishing.
3. Theoretical Astrophysics, Vol I: Astrophysical Processes, T. Padmanabhan, Cambridge University Press
4. Theoretical Astrophysics, Vol II:Stars and Stellar Systems, T. Padmanabhan, Cambridge University Press.
5. The Physical Universe: An Introduction to Astronomy, F. Shu, Mill Valley: University Science Books.
6. Textbook of Astronomy and Astrophysics with Elements of Cosmology, V. B. Bhatia, Pb-New Delhi, Narosa Publishing House.
7. The New Cosmos, A. Unsold and B. Baschek, New York:Springer Verlag.
8. The Physical Universe: An Introduction to Astronomy, F. Shu, Mill Valley: University Science Books.
9. Introduction to Cosmology, J. V. Narlikar, 3rd edition, Cambridge University Press.
10. Structure Formation in the Universe, T. Padmanabhan, Cambridge University Press.

Paper XV: Number theory in cryptography

Unit I

Introduction to Modular forms : Congruences Residue classes and complete residue system. Linear congruences. Reduced residue system and the Euler-Fermat theorem. Polynomials congruences modulo p, Lagrange's theorem. Simultaneous linear congruences, The Chinese remainder theorem, Application of Chinese remainder theorem, introduction to cryptography.

Unit II

Prime numbers, estimate of prime numbers, primality test, Polynomial congruences with prime power modulli, Fermat's little theorem and pseudoprime, Carmichael numbers, Wilson's theorem, Fermat-Kraitchik factorization method, Euler phi function and use of it in RSA cryptanalysis, Euler's generalization of Fermat's little theorem, modular exponentiation by repeated squaring method.

Unit III

Order of an integer modulo n, primitive roots for primes, composite numbers having primitive roots, theory of indices, application of primitive roots to cryptography.

Unit IV

Quadratic residues, Euler's criterion, Legendre's Symbol and its properties Gauss Law, the quadratic reciprocity law, Applications of reciprocity law. The Jacobi symbol and reciprocity law for Jacobi symbols. Applications of reciprocity law to Diophantine equations.

Books Recommended:

1. A course in number theory and cryptography, Neal Koblitz, Springer-Verlag.
2. An introduction to the theory of number, Ivan Niven, Zuckerman, Montgomery, willy India edition.
3. Elementary number theory, David M. Burton, , Tata McGraw Hill Edition.
4. Introduction to cryptography, Johannes A. Buchmann, Springer.

Syllabus for M.A./M.Sc.(Mathematics) Semester III (Elective Paper E3)

Elective Paper E3: Approximation Theory

Unit I

Different types of Approximations, Least squares polynomial approximation Weierstrass Approximation Theorem, Monotone operators, Markoff inequality, Bernstein inequality, Fejers theorem for HF interpolation.

Unit II

Erdos- Turan Theorem, Jackson's Theorems (I to V), Dini-Lipschitz theorem, Inverse of Jackson's Theorem, Bernstein Theorems (I,II, III), Zygmund theorem.

Unit III

Lobetto and Radau Quadrature, Hermite and HF interpolation, (0,2)-interpolation on the nodes of $\pi(x)$, existence, uniqueness, explicit representation and convergence.

Unit IV

Spline interpolation, existence, uniqueness, explicit representation of cubic spline, certain external properties and uniform approximation.

Book Recommended:

1. T.J. Rivlin ; An Introduction to the Approximation of functions, Dover Publications
2. E.W. Cheney: Introduction to Approximation Theory, McGraw-Hill Book Company
3. A. Ralston, A First Course in Numerical Analysis, MacGraw –Hill Book Company

Syllabus for M.A./M.Sc.(Mathematics) Course under Semester System

SEMESTER-IV

Compulsory Paper

Paper I: Lie Algebra

Unit I
Basic Concepts – definition and construction of Lie and associative algebras, algebras of linear transformations, derivations, inner derivations of associative and lie algebras, determinations of Lie algebras of low dimensionalities.

Unit II
Representations and modules, some basic module operations, Ideals, solvability, nil potency, extension of the base field.

Unit III
Solvable and Nilpotent Lie algebras- Weakly closed subsets of an associative algebra, nil weakly closed sets, Engel's theorem, Primary components, weight spaces.

Unit IV
Lie algebras with semi simple enveloping associative algebras, Lie's theorems, Applications to abstract Lie algebras, some counter examples, Universal enveloping algebras- definition and basic properties, The Poincare Birkhoff Witt theorem.

Books Recommended :
Lie Algebras – N. Jacobson, John Wiley

Paper II: Structures on odd dimensional differentiable manifolds, F-structure manifolds, Submanifolds

Unit I
Almost contact manifold, Lie derivative, affinely almost Co-Symplectic manifold.

Unit II
Almost Grayan manifold, almost Sasakian manifold, K-contact Riemannian manifold, Properties of curvature on these manifolds.

Unit III
Co-symplectic structure, F- structure manifold.

Unit IV
Submanifolds of almost Hermite manifolds and Kahler manifolds, Almost Grayan submanifolds.

Books recommended :
R.S. Mishra, Structures on a differentiable manifold and their applications. Chandrama Prakashan, Allahabad.

Paper III : Fluid Mechanics II

Unit I
Newton's law of viscosity, Nature of stress, Stress component in real fluid, Symmetry of stress tensor. Transformation of stress components. Stress invariants, Principal Stresses, Nature of Strain, Rates of strain components, transformation of rate of strain components, Rate of Strain Quadric, Relation between Stress and rate of Strain, Boundary conditions for viscous fluid.

Unit II
Navier-Stokes equation of motion-Conversation of momentum. Energy Equation-Conversation of Energy. Energy dissipation function. Energy dissipation due to viscosity. Diffusion of vorticity.

Unit III
Exact Solutions:
Plane Poiseuille and Couette flows between two parallel plates, Steady viscous flow through tubes of uniform crosss-section in form of circle, ellipse and equilateral triangle under constant pressure gradient. Flow between two co-axial cylinders and concentric spheres, unsteady viscous flow over a flat plate.

Unit IV
Reynolds number, slow viscous flow, flow past a sphere, Stoke's flow. Prandtl's Boundary layer concept, Boundary layer thickness-displacement, momentum of energy. Momentum and energy integrals, condition for separation, boundary layer flow along a semi-infinite plate in a uniform stream, Blasius solution.

Books Recommended :

1. G. K. Batchelor: An Introduction to Fluid Dynamics
2. Frank Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, New Delhi.
3. Z.U.A. Warsi: Fluid Dynamics, Theoretical and Computational approaches, C.R.C. Press
4. S.W. Yuan: Foundation of Fluid Mechanics, Prentice Hall of India Pvt. Ltd., New Delhi
5. L. Rosenhead, Laminar Boundary layer, Oxford Press
6. N. Curle and H J Davies: Modern fluid dynamics

Optional Papers for Semester-IV

Paper V: Special Functions and Orthogonal Polynomials

Unit I

Legendre Functions: Legendre's differential equation, Legendre polynomials, Laplace's integral for the Legendre Polynomials, Generating function, recurrence formulae, integral of a product of Legendre polynomials complete solution of Legendre's equation when n is an integer.

Unit II

Behaviour of Qn(z) at infinity,Integral formula for Qn(z) Heine's integrals for Qn(z), Neumann's integral for Qn(z), Heine's expansion of 1/(z-u) as a series of Legendre polynomials, Neumann's expansion theorem, Associated Legendre functions, Jacobi's Lemma, Integral representations of Pmn(z) and Qmn(z), addition-theorem for the Legendre polynomials.

Unit III

Elementary theory of Orthogonal polynomials: Introduction, Moment functional and orthogonality, Existence of OPS, Fundamental recurrence formula, Zeros, Gauss quadrature, Kernel Polynomials, Symmetric Moment functionals, related recurrence relations.

Unit IV

The Representation Theorem and Distribution Functionals: Introduction, Preliminary theorems (omitting proof of Helley's theorems), Representation theorem, Spectral points and zeros of orthogonal polynomials, determinacy of L in the bounded case.

Books Recommended:

1. E.T. Copson- Introduction to the theory of the functions of a complex variable.
2. T.S. Chihara- An introduction to orthogonal polynomials, Gordan & Breach.

Optional Paper VIII : Mathematical Epidemiology

Unit I

Historical asides of Epidemics, Simple Epidemic models: SI, SIR model, Basic Reproduction Ratio and practical applicatios.

Unit II

AIDS: Modeling the transmission dynamics of the HIV, HIV Modeling combination drug therapy, delay models for HIV infection with drug therapy, modeling the population dynamics of Acquired Immunity to parasite infection.

Unit III
Age dependent epidemic models and threshold criteria, Simple drug use epidemic model and threshold analysis, tuberculosis infection in Badgers and Cattle, derivation of diffusion equation, some exact solutions.

Unit IV
Virus dynamics: The basic model of virus dynamics, Antiviral drug therapy, Dynamics of Hepatitis B Birus, Dynamics of Immune response.

Paper X: Partial Differential Equations

Unit I
Introduction, basic concept and definition, classification of second order linear equation and method of characteristics, canonical form, Equations with constant coefficients, Supper position principle. Method of separation of variables.

Unit II
Boundary Value Problems, Maximum and Minimum Principles, Uniqueness and Stability theorem, Dirichlet problem for a Circle, Dirichlet Problem for a Circular annulus, Neumann problem for a Circle, Dirichlet problem for a Rectangular, Dirichlet problem involving Poisson equation.

Unit III
The Cauchy problem: The Cauchy problem, Cauchy-Kowalewsky Theorem, Hadmard example, Cauchy problem for homogeneous wave equations, Initial value problem, The Cauchy problem for Non-homogenous wave equation., The vibration string problem, Existence and uniqueness solution of the vibrating problem.

Unit IV
Fourier transform and Initial boundary value problems. Properties of Fourier Transform, Convolution (Fourier Transform), Step and impulse Function Fourier Transform, Semi-infinite region, Green's functions and boundary value problems.

Books Recommended:
1. Tyn Myint-U: Partial Differential Equations of Mathematical Physics, Elesveir Publication
2. I.N. Sneddon,: Elements of Partial Differential Equations, McGraw-Hill, 1988.

Paper XIII: Astrobiology
Prerequisite: This paper is open to students who studied Physics in UG.
Unit I:
Introduction to Astrobiology

- **A brief description of Astrobiology:** from Boyle and Lord Kelvin to space investigations**.**

- **Solar System Sites that might support life:** Mars, Titan, Europa. Also Enceladus, Io and Jupiter. Sites with the possibility of liquid water, complex chemistry and organic molecules.
- **The age of the Earth:** meteorites and the oldest rocks, early continents and oceans, early microfossils.
- **Life on Earth:** Life sustaining environments, evolution, origins, range, limitations, Chemistry/biochemistry.
- **Rise of life:** prebiotic reactions, an RNA world, the role of clays
- **Search for terrestrial planets:** detection techniques, searching for planetary atmospheres and chemical composition. Space missions and ground-based surveys.
- **Extraterrestrial biochemistry:** Alternatives to carbon and oxygen, other probable processes, constraints of physics, biology and chemistry on possibilities.
- **Space origins of life:** panspermia, comets as a source of biological material, life on Mars.
- **Environmental influences on life:** radiation, gravity, temperature, pressure, atmospheric composition.

Unit II

From Interstellar Molecules to Astrobiology/The Interstellar medium (ISM) and Prebiotic molecules

- **Interstellar Medium:** Definition and Composition of ISM, Formation of Interstellar Molecules, Organic Molecules in the Diffuse Interstellar Medium
- **Organic Molecules in Dense Clouds** -Organic Molecules Formed on the Grain Surface, Organic Molecules in Cold Dark Clouds, Organic Molecules in Hot Molecular Cores.
- Organic Molecules in Circumstellar Envelopes.
- Evidence for Specific Organic Molecules.
- Organic Molecules in Meteorites.
- The Relationship of Interstellar Molecules to the Origin of Life.
- **Comets:** Potential souces of Prebiotic Molecules for the Early earth: General description, Chemical composition, Origin and evolution of cometary matter, Delivary to the Earth. Comets & Meteorites, Cometary Dust and Refractory Organics, Cometary Volatiles.

Unit III: Basic Prebiotic Chemistry

- **Quantum Astrochemistry:** Definition, calculations approximation & application of Quantum chemistry to search the origin of life,
- **Methods of Computational Quantum Chemistry:** Application of Computational techniques: *ab initio* methods, Quantum Chemical methods etc.
- Introduction and usages of Gaussian Programme package.

Unit IV – Extraterrestrial Intelligence/Life

- **Origin of Life on Earth** - Theories about Origin of Life, Biochemical Origin of Life, Alternative Life, Urey-Miller Experiment, Timeline of Life & Mass Extinctions
- **Extrasolar Planets** – The Circum stellar Habitable Zone, The Inner limit and outer limit of HZ, Continuous Habitable Zone, Galactic Habitable Zone, Methods to Detect Extrasolar Planets & SuperEarths
- **Extraterrestrial Civilizations -** Fermi Paradox & Solutions, SETI - Search for Extraterrestrial Intelligence, Drake Equation, Rare Earth Hypothesis & Equation
- **Astrobiology Explorer Missions:** Astrobiology Space Exploration -Past, present and future missions: Space exploration of Comets, Space exploration of Titan, Mars Exploration
    - landers
    - sample return
    - space interferometers

Books Recommended:

1. Fundamentals of Molecular Spectroscopy - C.N. Banwell
2. An Introduction to Astrophysics- Baidyanath Basu.
3. Exploring Chemistry with Electronic Structure methods - James B. Foresman & Aeleen Frisch
4. Astrobiology: Future Perspectives- Pascale Ehrenfreund, William Irvine
5. Lectures in astrobiology Vol I - Muriel Gargaud, Bernard Barbier, Herve Martin & Jacques Reisse.
6. Lectures in astrobiology Vol II- Muriel Gargaud, Bernard Barbier, Herve Martin & Jacques Reisse.
7. Chemical evolution and Origin of Life- Horst Rauchfuss
8. Comets and the Origin and Evolution of Life- Paul J. Thomas, Christopher F. Chyba & Christopher P. Mckay
9. Life in Universe- Joseph Seckbach, Julian Chela Flores, Tobias Owen and Francois Raulin.

Paper XV: Cryptography

Unit I

Secure communication, cryptographic applications, Symmetric cipher model, Substitution technique: Ceasar cipher, Monoalphabatic cipher, Playfair cipher, Hill cipher, polyalphabatic cipher, one time pad, Transposition techniques, pseudorandom bit generator, linear feedback shift register sequences.

Unit II

Stream cipher and block cipher, simplified DES, Feistel cipher, DES, AES, S-box design, Boolean functions, bent functions, construction of finite fields, modular polynomial arithmetic.

Unit III

Public key cryptosystem, RSA cryptosystem, RAS and factoring, Rabin encryption , Key management, Diffie Hellman key exchange, discrete logarithm, ElGamal encryption, cryptographic hash function, message authentication codes, digital signature.

Unit IV

Factoring: p-1 method, quadratic sieve, discrete logarithm: DL problem, Shanks Babystep Giant step algorithm, Pollard pho algorithm, Pohlig-Hellman algorithm, Elliptic curve cryptography

Books Recommended:

1. Introduction to crypogrphy, Johannes A. Buchmann, Springer.
2. Cryptography and network security Principles and practices, William stallings, Pearson education.
3. Handbook of applied cryptography, Alferd J. Menezes, Paul C. Van Oorschot, Scott A. Vanstone, CRC press.
4. Introduction to cryptography and coding theory, Wade Trappe, Lawrance C. Washington.

Syllabus for M.A./M.Sc.(Mathematics) Semester IV (Elective Paper E4)

Elective Paper E4: Wavelet Theory

Unit I

Continuous Wavelets Transform: The Heisenberg uncertainty principle, the Shannon sampling theorem, Definition and examples of continuous wavelet transforms. A Plancherel formula, Inversion formulas, the Kernel functions, Decay of Wavelet transform.

Unit II

Inversion formulas, the Kernel functions, Decay of Wavelet transform, Frames: Geometrical considerations, Notions of frames.

Unit III

Discrete wavelet transform, signal decomposition (analysis), relation with filter banks, signal reconstruction, Multi resolution analysis, axiomatic description, the scaling function.

Unit IV

Construction of Fourier domain, Ortho normal wavelets with compact support: the basic idea, Algebraic constructions, Binary interpolation, Spline wavelets.

Books Reommended:

1. Christian Blatter, Wavelets: A Premier, AK Peters, 2002
2. C.K. Chui, An Introduction to Wavelets, Academic Press
3. Daubechies, Ten Lectures on Wavelets, SIAM Publication, Philadepphia
4. G. Kaiser, A friendly Guide to Wavelets, Birkhauser Boston, 1994